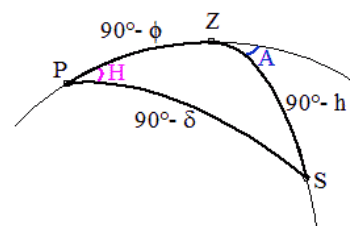


TriSph  
--o0o--  
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Yvon Massé  
[yvon.masse@sfr.fr](mailto:yvon.masse@sfr.fr)  
Translation – Fred Sawyer  
[fwsawyer@aya.yale.edu](mailto:fwsawyer@aya.yale.edu)



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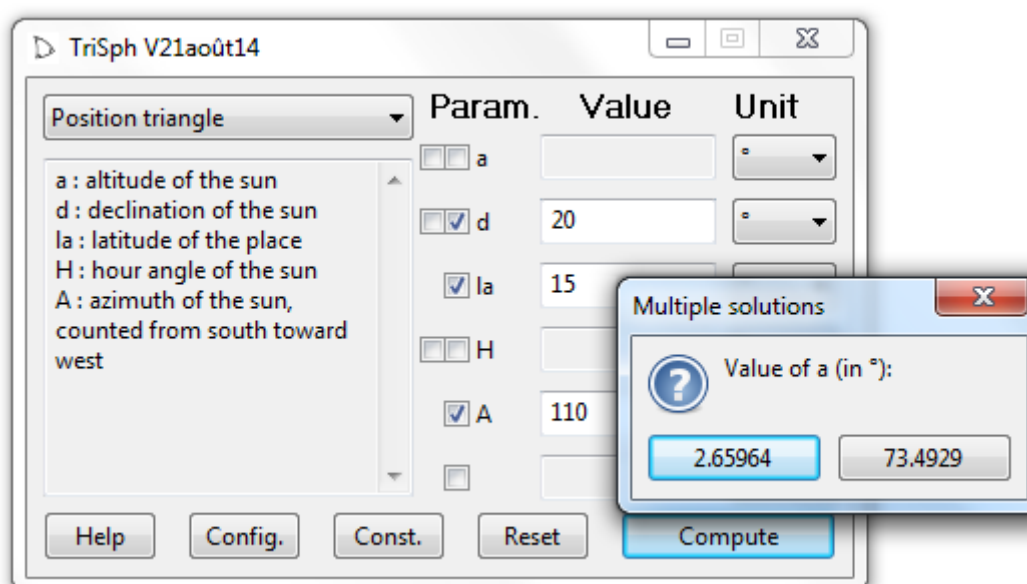
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## 1 Introduction

TriSph solves spherical triangles and plane triangles. A configuration file can be modified by the user (see § 6 p. 6) to assign symbols to different parameters of the triangle to display either their true value, a complementary value (summing to  $90^\circ$ ), or a supplementary value (summing to  $180^\circ$ ). A box is provided to document the various parameters.

Trisph comes with a configuration file including the principal spherical triangles used in gnomonics (see § 2.4 p. 4).

The V8oct12 version of TriSph runs under Windows 95/98/Me, NT/2000, XP, Vista and 7. From the V10nov12 version, TriSph is compiled under [Lazarus](#), a multiplatform compiler, which allows it to be ported to GNU / Linux and Mac OS X.

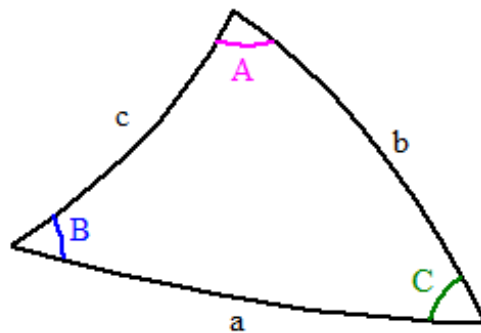


## 2 General notes

### 2.1 Spherical triangle

The spherical triangle is traced on a unit sphere. It consists of three vertices interconnected by three portions of a great circle. A great circle is a circle whose center corresponds to the center of the sphere.

The length of any side of the triangle is expressed in angular units, ranging from  $0^\circ$  to  $180^\circ$ . The angles at the vertices of the triangle are also between  $0^\circ$  and  $180^\circ$ . We can therefore measure 6 angular values on a triangle.



Spherical trigonometry shows that to compute all the values of a spherical triangle, we must know any three of the six sides or angles; this results in 20 cases. By permutation, we can reduce this number to 6 cases. Finally spherical trigonometry establishes, with the aid of the so-called polar triangle, a duality between the sides and angles, which allows us to divide this number by 2. The result is that, to solve a spherical triangle, we must know how to treat the following 3 cases:

- all 3 sides are known (a, b, and c)
- two sides and the angle between the sides are known (a, b and C)
- two sides and the angle at the end of one side are known (a, b and A)

### 2.2 Plane triangle

As for the spherical triangle, we can determine all the values of a plane triangle (usually just called a triangle, but here it is necessary to specify) from a minimum of values. Plane triangles do not have the side/angle duality specific to spherical triangles, but on the other hand, they have a well-known property which states that the sum of the angles is  $180^\circ$ .

In general, as for spherical triangles, we need to know three values - sides or angles - to determine the other values. The exception to this rule is when the three angles are known; only the ratio of the sides can then be established. In this case TriSph first proposes a solution with one of the sides set to 1m and then invites the user to replace the input of an angle by that of a side.

### 2.3 Practical applications

Spherical trigonometry is an essential mathematical tool in the following disciplines:

- astronomy
- gnomonics

- navigation
- geography

In spherical trigonometry, as we have seen, the angles and the sides are always between  $0^\circ$  and  $180^\circ$ ; the maximum size of the plane triangles is thus defined to be one half of the sphere.

In practice, to cover the entire sphere, it is necessary to combine two triangles. The transition from one to the other triangle is generally done by extending the value of the angles. Thus, for example the celestial sphere in gnomonics consists of Eastern and Western hemispheres; the angles (azimuth, hour angle, *etc.*) specific to the first are positive while those of the second are negative. The choice of the value of one of the angles thus establishes a specific triangle and hence also the values of the other angles.

Depending on the convention used, the extension of the angles may be either to negative values, as in the previous example, or to values greater than  $180^\circ$  as in the case of the azimuth as used by navigators which is counted from  $0^\circ$  to  $360^\circ$  from the north towards the east.

In TriSph the global variable Grot sets the work context:

Value of Grot	Work Context
0	Spherical or Plane Trigonometry
1	Practical Application: triangle 1
-1	Practical Application: triangle 2

## 2.4 Application to gnomonics

The sphere in gnomonics is the local celestial sphere. On this sphere, we can place the following 4 fundamental points:

- The north pole P
- Le zenith Z
- The Sun S
- The normal (perpendicular) to the dial K

We can thus trace 4 different triangles:

- PZS called the triangle of position: it makes the link between hourly coordinates (hour angle / declination) and local coordinates (azimuth / altitude).
- PZK: it allows us to calculate the parameters of the Equivalent Horizontal Dial.
- ZSK: it makes the link between the local coordinates and the coordinates on the dial. In addition, knowledge of these coordinates allows us to determine the orientation of the dial (gnomonic inclination / declination).
- PSK: it makes the link between the hourly coordinates and the coordinates on the dial.

Finally, the resolution of the triangle consisting of the points P and K as well as a side of  $90^\circ$  allows us to calculate the position of the hour lines.

### **3 Use right**

The V8oct12 version of TriSph is freeware. You can copy it, give it to your friends and use it freely.

From the V10nov12 version, TriSph is open source and passes under the MIT license (see License.txt file).

Despite all the care taken to verify this program, it is possible that some bugs have escaped my attention. If you find any malfunction, do not hesitate to let me know: I shall endeavor to make the necessary changes. I will also be very attentive to proposals from users for improving the program if they express a particular need.

### **4 Install/Uninstall**

The latest update of TriSph is available for download at:

[http://yvon.masse.perso.sfr.fr/trisph/index\\_en.htm](http://yvon.masse.perso.sfr.fr/trisph/index_en.htm)

#### **4.1 By installer in Windows**

The Windows Installer file was compiled by the freeware Inno Setup by Jordan Russell (<http://www.jrsoftware.org/>). After downloading the file, double-click on it and respond to the questions.

To uninstall TriSph, like all Windows applications, either enter Uninstall on the Start Menu or use the Windows Control Panel Add / Remove program configuration.

#### **4.2 Portable zipped version in Windows, GNU/Linux or Mac OS X**

You may run the program without installing it. Simply unzip the archive after downloading; the application is within the unzipped directory. With the aim to switch the language to English, edit the file TriSph.ini and, at the end of the file, remove the semicolon at the beginning of the line:

```
;Ext_lng=_en
```

The application launches when you double-click on it. To facilitate the launch, you may create one or more shortcuts.

To uninstall, simply remove the directory of the application and the eventual shortcuts.

#### **4.3 Source files**

Unzip the source files in a folder. Under Lazarus, open the project file TriSph.lpi and compile the program. The V21août14 version was compiled with the following configuration:

<b>OS</b>	<b>Lazarus Version</b>
Windows 7	0.9.30.4
Ubuntu 10.04	0.9.30-0
OS X	0.9.30.4RC3

## 5 Initialization file

At the launch of TriSph, the initialization file is used to define:

- the complementary part for the name of the files *Langage\*.lng*, *Lisezmoi\*.pdf* and *Config\*.cnf* to select a language
- the name of the configuration file. If it is different from *Config.cnf*, the previous complementary part is not applied.
- the program to use when the user clicks on the *Config.* button to edit the configuration file
- the radius to compute values in user units Au and Cu

The name of the initialization file must be *TriSph.ini* and it must be located:

- With Windows: in the folder from where *TriSph.exe* is launched
- With GNU / Linux or Mac OS X: in the application folder

If TriSph cannot find the file or if its content cannot be resolved, it uses the following default values:

- No complementary part of the file names (*cmp = ''*), French language
- Configuration File: ('TriSph' + *cmp* + '.cnf'), e.g. *TriSph\_en.cnf* if *cmp = '\_en'*
- Windows text editor: Notepad
- Text Editor GNU / Linux: Gedit
- Text Editor Mac OS X: TextEdit control with *open -e*
- User Radius: 1

## 6 Structure of the configuration file

The configuration file is a UTF8 encoded text file editable with a simple text editor. Its name is defined by the *TriSph.ini* file and it must be located:

- With Windows: in the folder from where *TriSph.exe* is launched
- With GNU / Linux or Mac OS X: in the application folder

It can be opened and edited with the *Config.* button.

The first line of the configuration file indicates the version with the word *TriSphX* where X is the version number. The absence of this line indicates that the version number is 0. The compatibility of different versions is noted in the table below.

Version of TriSph	Version of the configuration file
V8oct12	0
V10nov12	0
V13janv13	0 or 1
V11déc13	0 or 1
V21août14	0 or 1

The configuration file is then cut into sections by lines consisting of only three dashes (---).

The beginning of each paragraph should contain in order the following lines:

- The first is the title of the paragraph as it appears in the selection box of the application.
- The second lists in order the symbols of the sides and angles separated by commas. The first 3

should be the sides. The following 3 must be in the same order, the angles opposite the sides. Calling add-ons is also done in this line, see § 8 p. 8.

- The third is a 6-character string:
  - The first three correspond to the sides. If one of these characters is L, the triangle is considered plane. Otherwise the character O indicates that the side must be complemented at 90° and character N (or any other character) returns the result unchanged.
  - The last 3 characters correspond to the angles. The character O indicates that the angle must be supplemented to 180° and character N (or any other character) returns the result without change.
- The fourth is a 3-character string that defines the work context and possible values of the angles as shown in the table below.

	<b>P (or other)</b>	<b>Q</b>	<b>R</b>	<b>S</b>	<b>T</b>
<b>Trigonometry</b>	]0°, 180°[				
<b>Triangle 1</b>		]0°, 180°[	]0°, 180°[	]-180°, 0°[	]180°, 360°[
<b>Triangle 2</b>		]180°, 360°[	]-180°, 0°[	]0°, 180°[	]0°, 180°[

If there is at least one character different from Q, R, S or T in the string (only R or S in version 0 of the configuration file), the work context is spherical trigonometry and all angles must be between 0° and 180°.

- The fifth lists in order the symbols of the units separated by commas (only in version 1 of the configuration file). Warning: the symbols are case sensitive. In some cases, the use of upper or lower case corresponds to different units (see § 7.1 p. 7)
- The sixth lists in order the initial values separated by commas (only in version 1 of the configuration file).

The subsequent paragraph contains the text reported in the box of the application. Comments may be inserted only in this section. A comment line must start with a semicolon.

## 7 Miscellaneous

### 7.1 Units

<b>Symbol</b>	<b>Unit</b>
°	Degree. A right angle has 90°.
°°	Degree. Results are expressed in sexagesimal values in the form: degree:minute:second.
rad	Radian. A right angle is equal to $\pi/2$ rad
gr	Gradian. A right angle consists of 100 gr
m	Meter
Km	Kilometer = 1000 m. It is also the arc of one hundredth of a gradian on earth.
M	Nautical Mile = 1852 m. Arc of one minute of degree on earth. The nautical mile is to the degree as the kilometer is to the gradian.

Symbol	Unit
yd	Yard = 0.9144 m
mi	Mile = 1760 yd
Au	User Arc. Arc calculated with the user radius
Cu	User Chord. Chord calculated with the user radius
h	Hour. A right angle makes 6 h
hv	True hour. 12 h corresponds to 0°, 13 h to 15°, 14 h to 30°, <i>etc.</i> The values lower than 12 h correspond to negative values, so 11 h corresponds to -15°, 10 h to -30°, <i>etc.</i>

## 7.2 Entering values in degrees and hours

Angles in degrees can be entered either in decimal (e.g. 6.23° entered in this case as 6.23) or sexagesimal format (e.g. 6° 13' 48", one must then separate each value by :, *i.e.* take 6:13:48) or a mixture of both ... *i.e.* the degrees, minutes and seconds of arc can have decimal values. TriSph thus accepts the input: 6.1:6.2:96 and deduces from it the value 6.23°.

For angles in hours, the principle is the same.

## 7.3 Copy/paste values

All values of TriSph, whether input or output can be selected and copied to the clipboard. This possibility is very convenient to carry the values of the parameters from one instance of TriSph to another. This rule does not apply to Mac OS X which can only copy the values configured as input.

# 8 Complementary modules

## 8.1 Principle

The modules are independent programs that perform additional specific calculations on the angles of TriSph. Thus each side or angle of TriSph can be common with an additional module. The relevant value is then preceded by an additional check box in order to define it as input of the module. This check box also allows you to redisplay the window of the relevant complementary module when it is closed.

Param.	Value	Unit
<input checked="" type="checkbox"/> a	-0:50:33.36	°
<input checked="" type="checkbox"/> d	-16.6347	°
<input checked="" type="checkbox"/> la	49	°
<input checked="" type="checkbox"/> H	07:14:43	h

In the case shown here:

- a and d may be calculated by complementary modules or entered by the user and they are input to TriSph.
- H can be calculated by TriSph or entered by the user; it is also input of a complementary module.

The modules satisfy the same logic as TriSph; *i.e.* if a sufficient number of parameters are given, whatever they are, the module calculates the other parameters.

Calling the module is done by adding, in the configuration file, following the symbolic name concerned, the ~ character followed by a 2-character code defined by the following table:

Code	Module called	Angle
Ah	Solar ephemeris	Hour angle of the Sun
dc	Solar ephemeris	Declination of the Sun
hr	Solar altitude	Actual altitude of the Sun
st	Gnomonic projection	Site
Az	Gnomonic projection	Azimuth

Each module uses conventions, including sign, of its own. The configuration of TriSph must therefore be compatible with this convention for each common value with the module called.

## 8.2 Solar ephemeris

### 8.2.1 Introduction

This module solves the ephemeris of the Sun. The two parameters that can be shared with TriSph are the hour angle and the declination. The *Solar Ephemeris* also has the distinction of providing certain values, to the extent that they remain significant, even if all parameters necessary for the calculation are not given. This is for example the case of the declination of the Sun that can be given only from a date.

Param.	Value
Year	2013
<input type="checkbox"/> Day	
<input type="checkbox"/> Hour	
<input type="checkbox"/> T. zone	
<input type="checkbox"/> Longit.	

Unit: °

Reset Compute

The *Solar ephemeris* can in particular handle the following issues:

- Convert true time to civil time and vice versa
- Find a date depending on the declination of the Sun

Or some more original cases:

- Find the terrestrial longitude based solely on the declination and hour angle of the Sun
- Determine a date based on the equation of time

### 8.2.2 Sign convention

The *Solar ephemeris* module uses the following sign conventions:

- Time zone: value to add to UTC time to get standard time. In France, the time zone is set to 1 in winter and 2 in summer. In the eastern U.S., the time zone is -5 for standard time (winter) and -4 for daylight saving (summer).
- Longitude (terrestrial): positive west of Greenwich, negative east.

- Hour Angle: angle counted from the south meridian, positive in the afternoon, negative in the morning.
- Declination: positive for the northern hemisphere, negative for the southern hemisphere.

### 8.2.3 Time zone warning

The *Solar ephemeris* module considers only whole time zones between -12 and +12. Moreover, it does not take into account their correlation with longitude, so it can provide as solution the time zone 1 for the longitude of Washington, even though that makes no sense. Finally, no automatic function modifies the time according to the change of summer / winter time.

### 8.2.4 Entering the date and time of the computer

A star (\*) in the input field for the date or time allows you to enter respectively the date and time of the computer.

### 8.2.5 Ephemeris

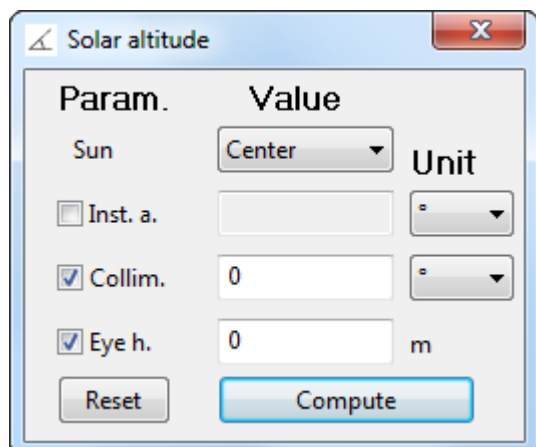
The *Solar ephemeris* module uses only one function, `ephe ()`, to calculate the ephemeris. This function gives the declination of the Sun and the equation of time as a function of universal time. All calculations of the module involve `ephe ()`, so that to increase the accuracy of the results one need only modify or supplement this function.

The current accuracy is around 1/100 of a degree over the period from 2010 to 2030, a little more over the period accepted by this module: from 1900 to 2099. The next step, for greater precision, is to introduce the nutation correction.

The `ephe ()` function is reproduced below to show how easy it is to obtain sufficient accuracy with a minimum of constants and calculations.

```
function ephe(jr, hr: Real; ts: Teph): Real;
const
  kl = 0.01720279; // Angular velocity of the mean longitude in radians/day
  jl = 43912.043;  // Central date of the period of use where the mean longitude is null
  km = 0.017202;   // Angular velocity of the mean anomaly in radians/day
  jm = 44199.46;   // Central date of the period of use where the anomaly is null
  e = 0.0167;      // Eccentricity of the terrestrial orbit
  ob = 0.4091;     // Obliquity of the ecliptic in radians
var
  ju, m, l, s: Real;
  x, y: Real;
begin
  ju := jr + hr/360;
  m := km*(ju - jm); l := kl*(ju - jl);
  s := l + 2*e*Sin(m) + 1.25*Sqr(e)*Sin(2*m);
  case ts of
    sLs: Result := RadToDeg(ArcTan2(Sin(s), Cos(s))); // Ecliptic longitude
    sDc: Result := RadToDeg(ArcSin(Sin(ob)*Sin(s)));  // Declination
    sAh0: begin
      x := Cos(s); y := Cos(ob)*Sin(s);
      Result := RadToDeg(ArcTan2(Sin(l)*x - Cos(l)*y, Cos(l)*x + Sin(l)*y));
    end;
  end;
end;
```

### 8.3 Solar altitude



The 'Solar altitude' window contains a table with two columns: 'Param.' and 'Value'. The first row has 'Sun' in the 'Param.' column and a dropdown menu showing 'Center' in the 'Value' column. To the right of this row is a 'Unit' label. Below the table are three rows of checkboxes: 'Inst. a.' (unchecked), 'Collim.' (checked), and 'Eye h.' (checked). Each checkbox is followed by a text input field. The 'Collim.' field contains '0' and has a unit dropdown showing '°'. The 'Eye h.' field contains '0' and has a unit dropdown showing 'm'. At the bottom are 'Reset' and 'Compute' buttons.

Param.	Value
Sun	Center
<input type="checkbox"/> Inst. a.	
<input checked="" type="checkbox"/> Collim.	0
<input checked="" type="checkbox"/> Eye h.	0

This module allows you to compute the actual altitude of the Sun to be used by TriSph as a function of the altitude, observed or measured (parameter *Inst. a.*), either of the center of the Sun or of its lower or upper edge.

Three corrections are included in this module:

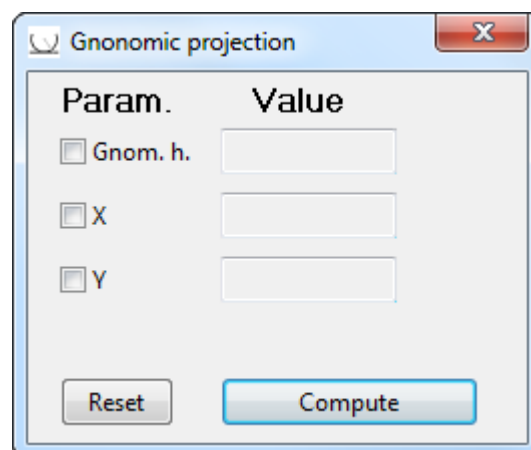
- the atmospheric refraction which depends directly on the observed height
- the collimation or error introduced by the measuring instrument, usually a sextant. The collimation (parameter *Collim.*) is added to the value read on the instrument
- the depression of the horizon: at sea the visual horizon, which serves as a reference for measuring altitude, subsides when the eye of the observer rises above the water. This correction, which depends on the parameter *Eye h.*, must be expressed in meters.

When the parameters *Collim.* and *Eye h.* are zero, which is the case when the module is called, only the refraction correction is applied.

### 8.4 Gnomonic projection

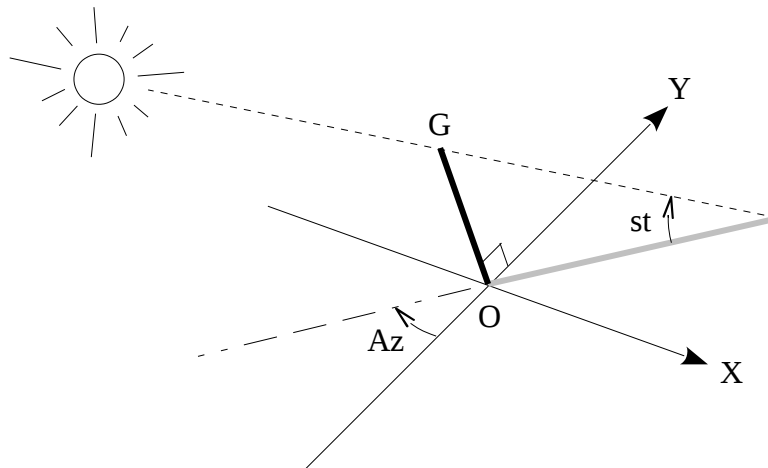
#### 8.4.1 Introduction

Mathematically, the gnomonic projection is the central projection of a sphere onto a plane, the projection center being the center of the sphere. Dialists commonly use this projection on a plane dial when they draw lines that will be covered by the shadow of a peephole or the end of a gnomon; these accessories actually embody the projection center. The *Gnomonic projection* module fits into this practice: it maps the coordinates of the sphere, site (st) and azimuth (Az), with the Cartesian coordinates on the plane, X and Y, according to the graphic below. The parameter *Gnom. h.* corresponds to the length OG.



The 'Gnomonic projection' window contains a table with two columns: 'Param.' and 'Value'. There are three rows of checkboxes: 'Gnom. h.' (unchecked), 'X' (unchecked), and 'Y' (unchecked). Each checkbox is followed by a text input field. At the bottom are 'Reset' and 'Compute' buttons.

Param.	Value
<input type="checkbox"/> Gnom. h.	
<input type="checkbox"/> X	
<input type="checkbox"/> Y	



### 8.4.2 Sign conventions

The *Gnomonic projection* module considers only the strictly positive site angles, *i.e.* those above the plane of projection. The azimuth is counted positive towards negative X, as in the figure above, and negative towards positive X.

The unit of the parameters *Gnom. H.*, *X* and *Y* is the choice of the user, but it must be the same for all these parameters.

## 9 Resolutions of classic problems

The goal of this chapter is to show the flexibility and the didactic interest of TriSph for solving various problems involving the Sun's position.

### 9.1 Hour of the Sun's meridian passage (noon)

This hour is used by gnomonists to determine the north/south direction with great precision, much better than the indication of a compass even if it is corrected by the magnetic declination. Only the *Solar ephemeris* module is necessary to obtain this hour. It can be activated with, for example, the configuration *System ecliptic/equatorial*.

Set the value of the hour angle to 0; then select:

- the day
- the current time zone
- the longitude of the place

The hour sought is given by pressing the *Compute* button.

Param.	Value	Unit
Year	2013	
<input checked="" type="checkbox"/> Day	11/8	Nov 8
<input type="checkbox"/> Hour	12:35:45	12:35:45
<input checked="" type="checkbox"/> T. zone	1	
<input checked="" type="checkbox"/> Longit.	-2	°
<input checked="" type="checkbox"/> H. angle	0	°
Reset		Compute

## 9.2 Astronomic Point of the Sun

Knowledge of the altitude of the Sun at a given instant allows us to define on the surface of the earth a circle upon which the observer is situated. By considering only a small local part of this circle, we can treat it as a line which, in the classic method of determining a point at sea, is called the *line of position*. Moreover, this method is a good mix of calculation and plotting to set this line on a map with the peculiarity of initiating the procedure by an approximate position (for more details, see this page: [http://en.wikipedia.org/wiki/Intercept\\_method](http://en.wikipedia.org/wiki/Intercept_method)). The use of TriSph will however let us solve this problem by a different method which, in principle, will allow us to compute all the points of the circle. Always, by being interested only in 2 points close to the estimated position, we will be able to trace the line of position.

The configuration used is *Position triangle*.

### 9.2.1 True altitude of the Sun

Select *Lower edge* and set in the *Solar altitude* module:

- the measured height
- the collimation of the sextant
- the height above the water at which the measurement is made

and then press the *Compute* button. The value of the Sun's altitude will appear in the main window.

The screenshot shows the 'Solar altitude' window with the following settings:

Param.	Value	Unit
Sun	Lower edge	
<input checked="" type="checkbox"/> Inst. a.	49:59	°
<input checked="" type="checkbox"/> Collim.	-0:2	°
<input checked="" type="checkbox"/> Eye h.	3	m

Buttons: Reset, Compute

The screenshot shows the 'Solar ephemeris' window with the following settings:

Param.	Value	Unit
Year	2013	
<input checked="" type="checkbox"/> Day	8/8	Aug 8
<input checked="" type="checkbox"/> Hour	16:16:4	16:16:04
<input checked="" type="checkbox"/> T. zone	2	
<input checked="" type="checkbox"/> Longit.	1:40	°

Buttons: Reset, Compute

### 9.2.2 Calculation of the hour angle of the

#### Sun

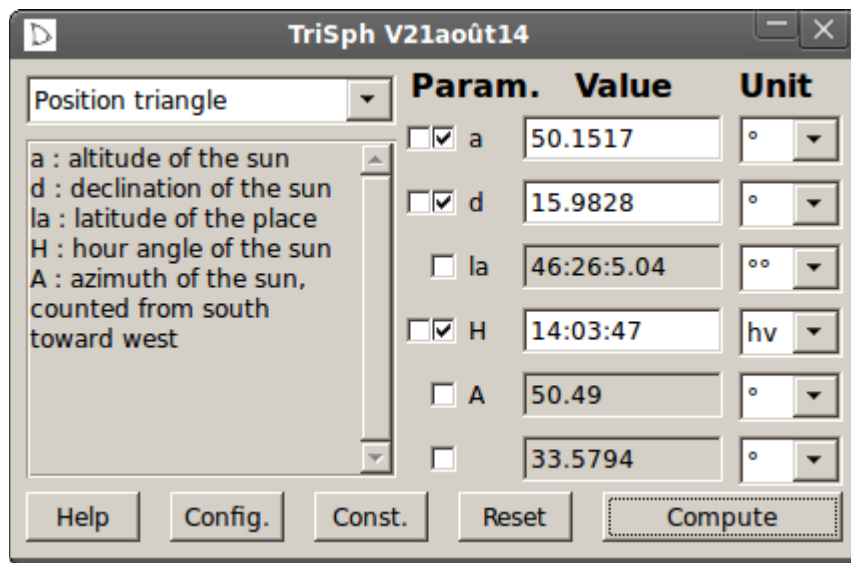
In the *Solar ephemeris* module, assign:

- the day
- the time of the measurement
- the time zone
- a longitude close to the point where one is supposed to be

Then press the module's *Compute* button. The values of the Sun's declination and hour angle will appear in the main window.

### 9.2.3 Calculation of the latitude

In the main window of TriSph, select as input the results obtained above and press the *Compute* button. Two latitudes are proposed; they correspond to the intersections of the circle and of the meridian of the selected longitude. Choose the latitude which corresponds to the estimated point. With the longitude, we obtain a first point on the map. By introducing a different longitude and redoing the calculations, we obtain a new latitude which will give a second point. We can thus trace the line of position that passes through these 2 points.



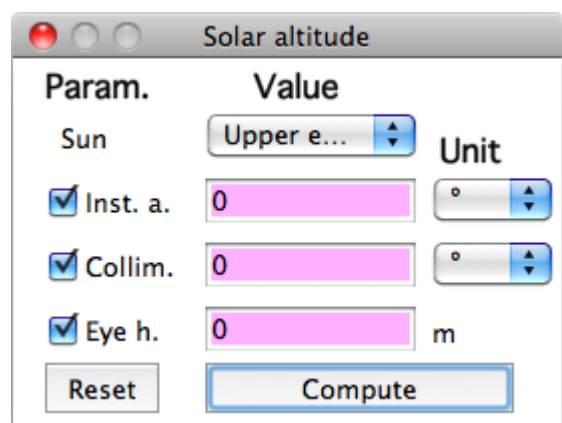
### 9.3 Hours of rising and setting of the Sun

To have a precise value (and completely theoretical since in practice two fundamental parameters are not controlled: the height of the horizon and the atmospheric refraction for low altitudes, this latter is strongly linked to meteorological conditions), we must proceed by iteration. We will define, in the first case, the actual altitude of the Sun at the instant of setting or rising. With the date, we will then determine an approximate value of the declination, this will then allow us to define an approximate hour to then fix the declination and so on. These different steps are detailed below, the configuration used is *Position triangle*.

#### 9.3.1 Actual altitude of the Sun

We generally define the instants of rising or setting of the Sun as the appearance or the disappearance of its upper edge on the horizon, this last being considered the same as the horizontal plane.

To take this condition into account (the user can always define his/her own condition) and compute the actual altitude of the Sun we must select *Upper edge* in the *Solar altitude* module and set all the values to 0. Then press the module's *Compute* button.



Param.	Value	Unit
Year	2013	
<input checked="" type="checkbox"/> Day	11/8	Nov 8
<input type="checkbox"/> Hour		
<input checked="" type="checkbox"/> T. zone	1	
<input checked="" type="checkbox"/> Longit.	-2	°

Reset Compute

### 9.3.2 Approximate declination

In the *Solar ephemeris* module, set:

- the date
- the current time zone
- the longitude of the place

and press the module's *Compute* button.

### 9.3.3 Approximate hour

In the main window of TriSph, set the latitude of the place and enter the previously obtained results into TriSph, then press the *Compute* button. Choose the value of H (hour angle) as a function of the time to be calculated: sunrise or sunset. Then select the result of the hour angle as the input of *Solar ephemeris* module and press the *Compute* button of this module to get the approximate time.

Param.	Value	Unit
<input checked="" type="checkbox"/> a	-0.8426	°
<input checked="" type="checkbox"/> d	-16.6347	°
<input checked="" type="checkbox"/> la	49	°
<input checked="" type="checkbox"/> H	07:14:43	hv
<input type="checkbox"/> A	-65.1986	°
<input type="checkbox"/>	-38.4303	°

Help Config. Const. Reset Compute

### 9.3.4 Iteration

Alternately press the *Compute* buttons of the main window of TriSph and of the *Solar ephemeris* module. After 2 to 3 cycles, the values will no longer change; this indicates that the definitive result has been attained. The time of rising/setting is found in the *Solar ephemeris* module.

#### 9.4 Inclination/declination of the plane surface

The gnomonic inclination and declination of a plane surface are the 2 fundamental parameters that determine its orientation and are used to calculate the sundial that will be installed. Many methods have been proposed to measure these parameters independently; spherical trigonometry, however, shows that we can obtain them at the same time if we can determine the same direction expressed in the local coordinate system (site/azimuth) and in the coordinate system of the plane surface ("altitude" above the surface / direction relative to the line of greatest slope).

The direction that comes naturally to the mind of the gnomonist is that of the Sun. On the one hand, he knows how to calculate it in the local coordinate system, and on the other, he commonly uses a tool called a planchette that permits him to measure the direction of the Sun in the system of the plane by considering the position of the shadow of a gnomon.

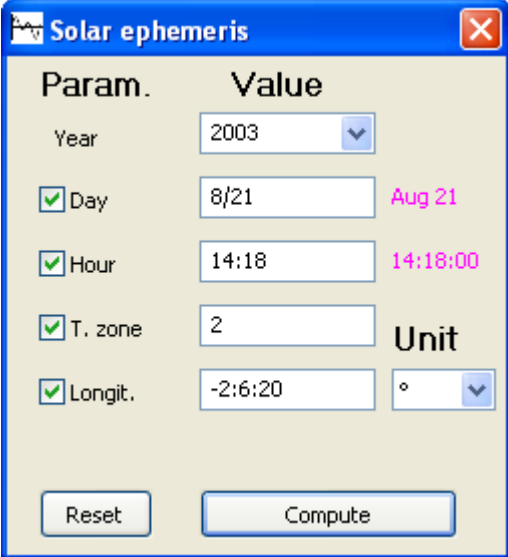


*Use of the planchette:*

- *The X axis must be horizontal, the line from O to the nodus must be perpendicular to the planchette.*
- *Determine the position of the shadow of the nodus in the OXY coordinate system and the corresponding hour.*

### 9.4.1 Direction of the Sun in the local system

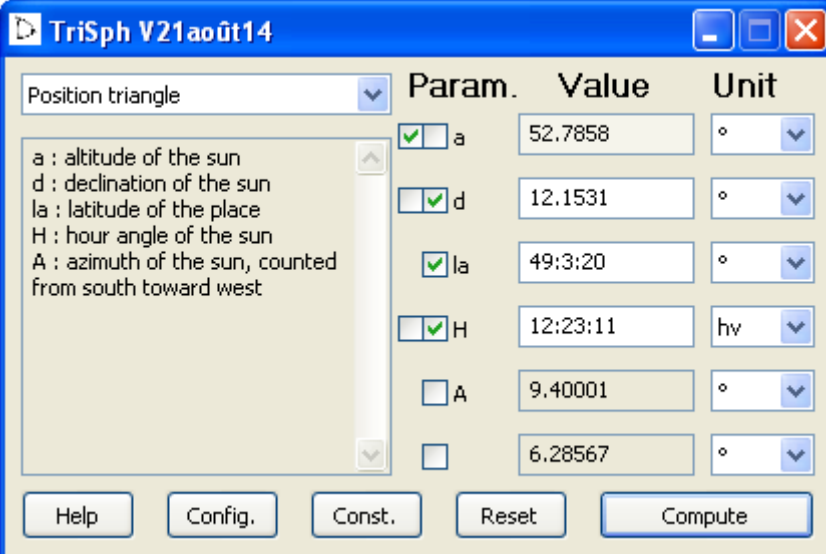
Select the configuration *Position triangle*. In the *Solar ephemeris* module, set all the fields and press the *Compute* button.



Param.	Value	Unit
Year	2003	
<input checked="" type="checkbox"/> Day	8/21	Aug 21
<input checked="" type="checkbox"/> Hour	14:18	14:18:00
<input checked="" type="checkbox"/> T. zone	2	
<input checked="" type="checkbox"/> Longit.	-2:6:20	°

Reset Compute

In the main window, enter the latitude and select the declination and the hour angle as inputs. Then press the *Compute* button. We thus obtain the azimuth and the actual altitude of the Sun. To obtain the apparent altitude, select the altitude as input for the *Solar altitude* module.



Position triangle

- a : altitude of the sun
- d : declination of the sun
- la : latitude of the place
- H : hour angle of the sun
- A : azimuth of the sun, counted from south toward west

Param.	Value	Unit
<input checked="" type="checkbox"/> a	52.7858	°
<input type="checkbox"/> d	12.1531	°
<input checked="" type="checkbox"/> la	49:3:20	°
<input checked="" type="checkbox"/> H	12:23:11	h
<input type="checkbox"/> A	9.40001	°
<input type="checkbox"/>	6.28567	°

Help Config. Const. Reset Compute

Param.	Value	Unit
Sun	Center	
<input type="checkbox"/> Inst. a.	52.7984	°
<input checked="" type="checkbox"/> Collim.	0	°
<input checked="" type="checkbox"/> Eye h.	0	m

Reset Compute

In this module, if necessary select the center of the Sun and set the collimation and height of the eye to 0. Then press the *Compute* button and copy the result into the clipboard.

#### 9.4.2 Direction of the Sun in the dial's reference system

Open a new instance of TriSph and select the *System horizontal/dial* configuration. In the main window, click on the Solar altitude as input and paste the value calculated earlier.

Then fill in the values from the planchette in the *Gnomonic projection* module and press the module's *Compute* button.

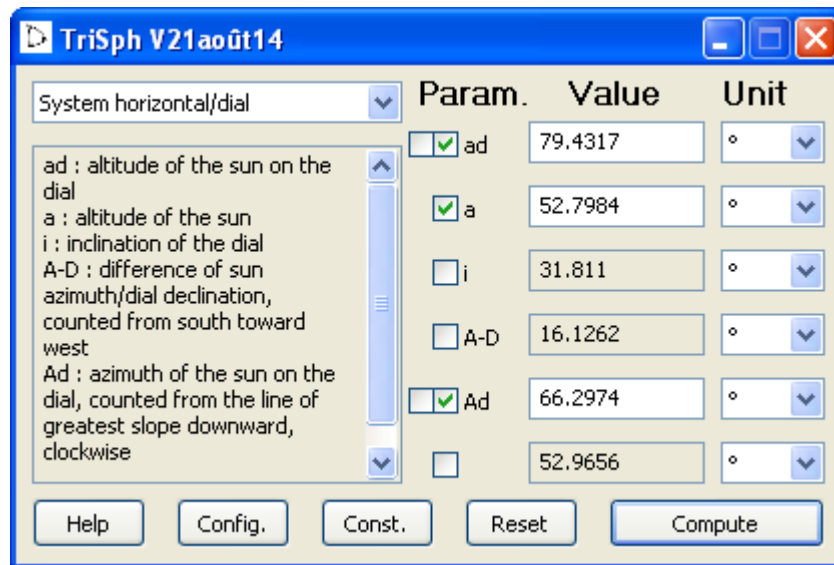
Param.	Value
<input checked="" type="checkbox"/> Gnom. h.	120
<input checked="" type="checkbox"/> X	20.5
<input checked="" type="checkbox"/> Y	9

Reset Compute

In the main window, select as input the results obtained in the previous step (ad and Ad) and press the *Compute* button. TriSph then provides directly the inclination of the plane surface; however, in some cases, two values of inclination may be offered: then choose the appropriate value. Declination is obtained by taking the difference between the azimuth of the Sun A calculated in the previous chapter and the result A-D (D being the declination of the wall, we have  $D = A - (A - D)$ ). In this example we get:

$$D = 9.40001 - 16.1262 = -6.72619$$

A declination of  $6.73^\circ$  to the east.



## 9.5 Islamic Asr prayer time

According to the Shafi'i school, the beginning of Asr is the moment when the shadow of a person is equal to the length of his shadow at the culmination of the Sun (noon) plus his own height. Considering this precept in a purely astronomical context in which the shadow is cast on a perfectly horizontal plane, we can calculate this moment with great precision. The different steps that need to be followed are the following:

- Calculate the actual height of the Sun at its culmination
- Calculate its apparent height taking refraction into account
- Calculate the length of the shadow for a unit height
- Calculate the apparent height of the Sun when the shadow length increases by one unit
- Calculate the actual height of the Sun at the beginning of Asr
- Calculate the corresponding time. We will use the iterative method described for calculating sunrise / sunset (see § 9.3 p. 14)

### 9.5.1 Altitude of the Sun at culmination

Select the *Position triangle* configuration and, in the *Solar ephemeris* module, assign values for:

- the date
- the time zone
- the longitude of the place

The screenshot shows the 'Solar ephemeris' window with the following parameters and values:

Param.	Value	Unit
Year	2013	
<input checked="" type="checkbox"/> Day	12/11	Dec 11
<input type="checkbox"/> Hour		
<input checked="" type="checkbox"/> T. zone	1	
<input checked="" type="checkbox"/> Longit.	-2:20:23	°

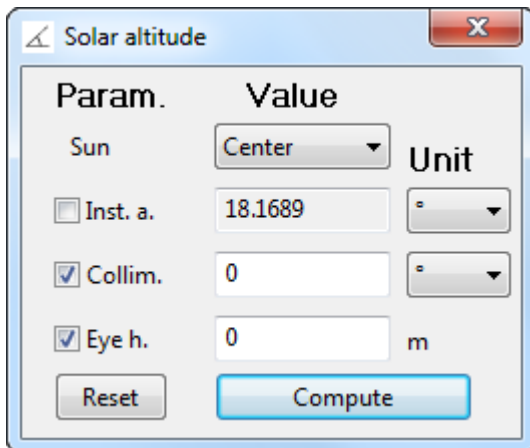
Buttons: Reset, Compute

In the main window, select the hour angle as the input of both TriSph and the *Solar ephemeris* module; then give a value slightly greater than 12 hv (TriSph cannot accept 12 hv or 0 hv which are not in the field of definition of spherical trigonometry). Press the *Compute* button of the *Solar ephemeris* module which then gives the declination of the Sun. Select this value as input for TriSph and set the latitude. Then press the *Compute* button to obtain the real altitude of the Sun and select this value as input for the *Solar altitude* module.

The screenshot shows the 'TriSph V21aout14' window with the 'Position triangle' configuration selected. The parameters and values are as follows:

Param.	Value	Unit
<input checked="" type="checkbox"/> a	18.1191	°
<input type="checkbox"/> d	-23.0317	°
<input checked="" type="checkbox"/> la	48:50:57	°
<input checked="" type="checkbox"/> H	12:0:0.1	hv
<input type="checkbox"/> A	0.0004	°
<input type="checkbox"/>	0.00029	°

Buttons: Help, Config., Const., Reset, Compute



**Solar altitude**

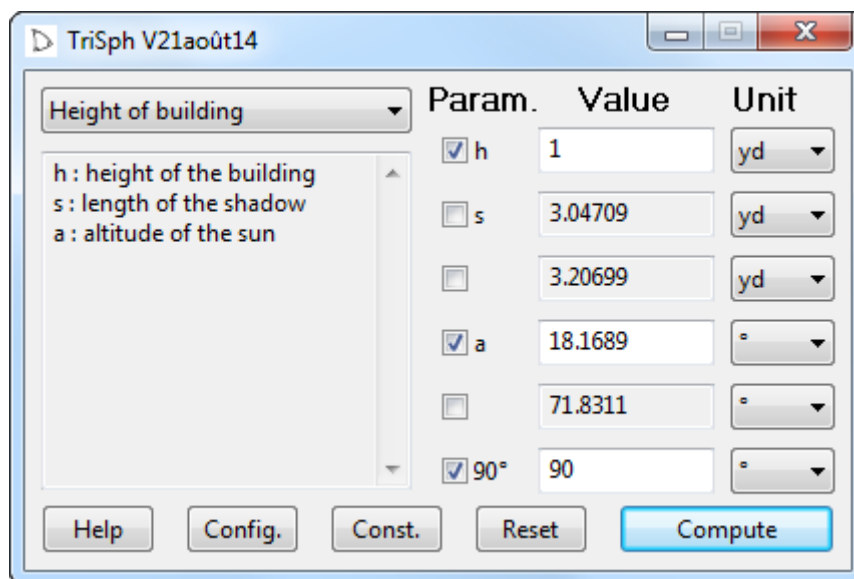
Param.	Value	Unit
Sun	Center	
<input type="checkbox"/> Inst. a.	18.1689	°
<input checked="" type="checkbox"/> Collim.	0	°
<input checked="" type="checkbox"/> Eye h.	0	m

Reset Compute

In this module, if necessary select the center of the Sun and set the parameters *Collim.* and *Eye h.* to 0. Press the *Compute* button to get the apparent altitude of the Sun. Select and copy the result into the clipboard.

### 9.5.2 Length of the shadow at culmination

Open a second instance of *TriSph* and select the configuration *Height of building*. Paste the apparent height of the Sun and enter the value 1 for the height of the building. Press the *Compute* button to get the length of the shadow at the culmination of the Sun.



**TriSph V21a0t14**

Height of building

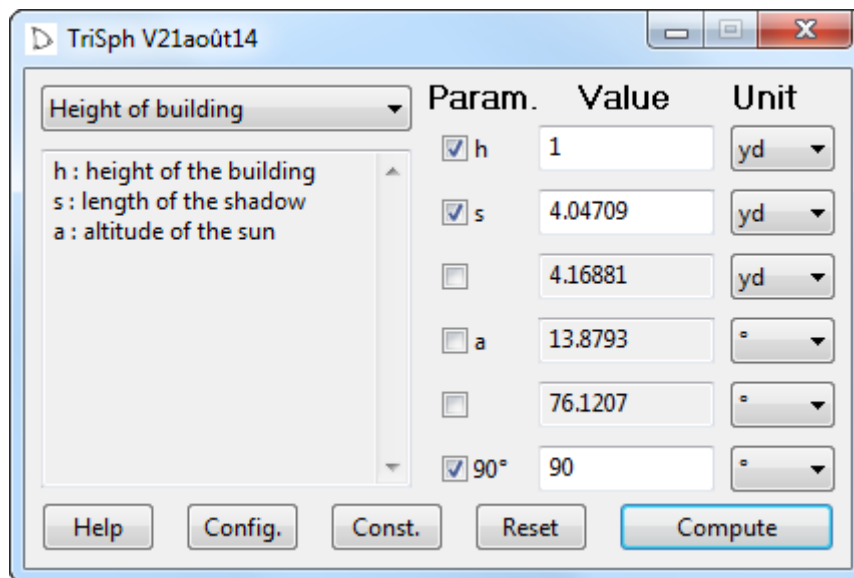
h : height of the building  
s : length of the shadow  
a : altitude of the sun

Param.	Value	Unit
<input checked="" type="checkbox"/> h	1	yd
<input type="checkbox"/> s	3.04709	yd
<input type="checkbox"/>	3.20699	yd
<input checked="" type="checkbox"/> a	18.1689	°
<input type="checkbox"/>	71.8311	°
<input checked="" type="checkbox"/> 90°	90	°

Help Config. Const. Reset Compute

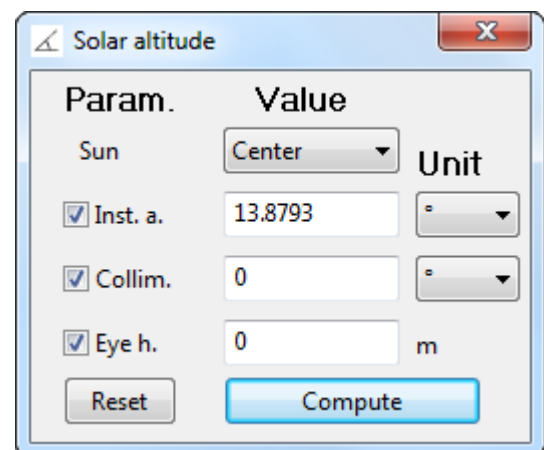
### 9.5.3 Apparent altitude of the Sun at beginning of Asr

Select the length of the shadow as input and increase its value by one unit. Switch the altitude of the Sun as result and compute the triangle. We get the apparent altitude of the Sun at the beginning of Asr. Select and copy this value in the clipboard.



#### 9.5.4 Time of the beginning of Asr

In the Solar altitude module already open, select the parameter *Inst. a.* as input and paste the apparent altitude of the Sun.



In the main window, change the settings by selecting:

- a as a result of the *Solar altitude* module and as input for TriSph
- H as a result of TriSph and as input for the *Solar ephemeris* module

Then press the *Compute* button of the *Solar altitude* module to obtain its real altitude. Then press the *Compute* button in the main window and select the afternoon hour angle.

Param.	Value	Unit
<input checked="" type="checkbox"/> a	13.8138	°
<input checked="" type="checkbox"/> d	-23.0317	°
<input checked="" type="checkbox"/> la	48:50:57	°
<input checked="" type="checkbox"/> H	13:53:04	h <sup>m</sup> s
<input type="checkbox"/> A	26.6692	°
<input type="checkbox"/>	18.7197	°

Param.	Value	Unit
Year	2013	
<input checked="" type="checkbox"/> Day	12/11	Dec 11
<input type="checkbox"/> Hour	14:36:59	14:36:59
<input checked="" type="checkbox"/> T. zone	1	
<input checked="" type="checkbox"/> Longit.	-2:20:23	°

Then press alternately the *Compute* buttons of the *Solar ephemeris* module and of the main window until the values no longer change. We then obtain the start time of Asr.

## 10 History of the versions

Version 21 August 2014 :

- Multilingual version
- Management of the unit list from the .lng file
- New imperial units: yd and mi
- Correction of minor bugs
  - Error in calculations of altitude corrections
  - Input of user radius strictly positive
  - Error of unit for some messages of ResEph

Version 11 December 2013:

- Addition of the modules Solar ephemeris, Solar altitude and Gnomonic projection
- Solution of plane triangles

- Modification of the configuration file to make use of the preceding new items
- Use of an initialization file .ini
- Introduction of the documentation with detailed description of complex solutions
- Improved selection box for multiple solutions
- Four new units
- Introduction of the user radius for units Au and Cu
- Compilation for GNU/Linux and OSX
- Correction of minor bugs
  - Misinterpretation of sexagesimal values between  $0^\circ$  and  $-1^\circ$
  - Search path of the configuration file under GNU/Linux

Version 13 January 2013:

- Selectable units
- Configurable units and initial values
- Management of angles from  $180^\circ$  to  $360^\circ$
- Proper management of boundary values
- Display rounded values
- Two new triangles in the configuration file
- Correction of minor bugs
  - Sexagesimal format
  - Selection box for triangles non-editable
  - Windows installation for editing the configuration file

Version 10 November 2012:

- Compiled under Lazarus
- Adaptations for functioning under GNU/Linux
- Creation of this file in pdf format
- Simplification of the solution a, b and A
- Minor corrections

Version 8 October 2012:

- Initial version